**Ballistic Velocity Variance**

**Prepared for**

**Rabbit Creek Shooting Club**

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**Introduction**

We want to start by saying thank you to the Rabbit Creek Shooting Club (RCSC) for choosing DRT Consulting LLC (DRT) for this study. RCSC serves the community of Anchorage Alaska by educating youth and adults in firearms safety and recreation (Figure 1 shows the sign of RCSCs’ home range off the Seward Highway outside of Anchorage). Since DRT started serving the Anchorage community 13 years ago, we have had the privilege of providing statistical consulting for many different sporting events. However, this is the first time we have been asked to provide services related to ballistics and we are excited to show you what we have found.

RCSC is hosting a rifle competition where each shooter will make a series of 500-yard shots. Each shooter will be firing the same make and model rifle with the same factory ammunition. RCSCs’ goal is to test the shooters’ ability to sight in and operate a rifle not their own while firing standard ammunition they may or may not be familiar with. RCSC has requested DRT to test the 4 available ammunitions to determine which one has the minimum variance in velocity. This is due to RCSCs’ belief that minimizing velocity variance will minimize the variance in bullet drop. The magnitude of the drop is not as important as the consistency of the drop. Bullet drop refers to where the bullet strikes the target on the y-axis. This minimizing will provide each competitor a fairer chance of being judged only on their skill as a shooter and not subject to the inconsistencies of the ammunition.

DRT has investigated the relevance of velocity as the primary contributor to drop and found it to be true. DRT has also identified the ammunition that provides the minimum variance to be ammunition B: ***American Eagle 168gr OTM***with a mean velocity of **2,406 ft/s** and a variance of **300 ft/s** (standard deviation = 17.3 ft/s). We also found that the velocity variance may be significantly different from rifle to rifle. One other factor, ‘*shots fired since last cleaning*’ was found to be insignificant in this test.

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Figure : Rabbit Creek Shooting Park. (https://www.adfg.alaska.gov/index.cfm?adfg=anchoragerange.main)

**Does Velocity Matter?**

We wanted to first investigate the importance that velocity plays in trajectory. Is there a legitimate basis for pursuing the variance in velocity? Projectile trajectories are governed by three main factors, Gravity (g), Velocity (v), and Drag (). For the purpose of this study, gravity is assumed to be a constant (g = 32.2 ft/s^2). Velocity and drag then are the varying components of a projectiles’ path. When we initially approached this project, we assumed that a simple formula could be found to relate distance traveled, flight time, etc. However, solving for projectile motion requires a rigorous application of differential equations. A simplified formula for the distance traveled by a projectile starting parallel to the ground and assuming it “never” hits the ground is:

Even this simplified formula for distance is daunting considering all bullets will eventually hit the ground, the velocity is changing over time, and drag is not linearly related to velocity above the speed of sound (1,124 feet/s).

There are several free trajectory calculators that can be downloaded to alleviate the burden of doing trajectory calculations yourself. We decided to use a free software, ‘PointBlank Ballistics’ (PBB), the download can be found here: <https://www.huntingnut.com/pointblank.html>). The software is straightforward, and the complications of differential equations are abstracted away. Figure 2 shows the graphical user interface. All other calculations and manipulations were performed in R (<https://cran.r-project.org>) and formatted MS Excel.

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Figure : PointBlank Ballistics GUI.

The inputs of interest are Ballistic Coefficient (BC), Velocity (Vel), Weight (Wt), Altitude (Alt), Zero Distance (constant at 100-yards), and Temperature (Tmp, °F). Published values for ammunition BC, Vel, Wt can be found on the manufacturer’s website or with retailers. Table 1 has the published values for each of our tested ammunitions. The values of Alt=200ft, Tmp=30°F were actual range day values.

To test the effect of changing input values on drop we set a ‘Base’ equal to the ammunitions published values (Table 2 has the values used for the test, ammo ‘A’). Then each input was varied by +/- 10%. We created a table of all permutations of these values. This was done for the 3 values of each of the 5 inputs equaling = 243 possible combinations. We selected a random sample of 20 from this table and used PBB to calculate bullet drop at 500-yards. Table 3 has the complete table for analysis of inputs on bullet drop.

Table : Published specifications for selected ammunition.



Table : Ammunition 'A' Input Parameters for test.



Table : Sampled data from Ammo 'A' permutation table.



The sampled data revealed the impact of each of the inputs on the bullet drop. Figure 3 shows the impact of Velocity and BC on bullet drop. Drop is inversely proportional to both Velocity and BC. Another observation is Velocity and BC do not appear to be independent. As the Velocity increases the effect of BC appears to decrease.

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Figure : Effect of Velocity and BC on Bullet Drop calculation.

We tested the significance of each input using a linear model and a method called Analysis of Variance (ANOVA). These are powerful statistical tools that allow us to make decisions on what model will be best for our analysis. We started out by assuming that:

1. Bullet drop is equal to a linear combination of all the inputs.
2. All of the errors are normally distributed.
3. The variance is constant.

There are several tests available to ensure each of these assumptions holds true. For our purposes, we will let the assumptions stand. Below is the linear model we incorporated:

Each of the β’s are coefficients for the independent variables. is the intercept term. Notice also that we are assuming an interaction (Vel \* BC) between velocity and the ballistic coefficient. When we run the model in R (using the lm() function) our coefficients are found to be:

Now that we have the linear model, we can find which variables are “significant” using ANOVA. For our purposes, we want to be 99% confident that a variable is informative before we employ it in our model (we always want the most *parsimonious* model as it provides the best answer at the lowest cost). ANOVA begins with a Hypothesis that none of the inputs to the model matter. Then for each input we proceed to get the Degrees of Freedom (Df), Mean Squared Error (MSE), F-Value, and the F distributions tail probability given the degrees of freedom of the input and the residuals. This process can be challenging to understand. A web search of ANOVA is a great start to understanding the workflow. I suggest starting at the site Guru99 (<https://www.guru99.com/r-tutorial.html>) and selecting the R ANOVA Tutorial under the Data Analysis section.

Table 4 is the ANOVA summary table. Observe in the ANOVA table that we do not have sufficient evidence to conclude that Velocity, Ballistic Coefficient and their interaction are insignificant at the 99% level. **Therefore, pursuing the minimization of velocity matters.** While the pursuit of minimizing the variance of the Ballistic Coefficient would also be valuable, that is outside the scope of this study. We recognize the remaining inputs are important in a thorough study of projectile motion but given the data we have and the 99% level set, they are insignificant.

Table : ANOVA table for test of significance of model inputs.



**Data Gathering**

RCSC’s request is test the 4 available ammunitions to determine which one has the minimum variance in velocity. Ballistic velocity can be measured with a Ballistic Chronograph. I acquired a *V3 Barrel Mounted Ballistic Chronograph* (Figure 4) from the RCSC team for use in the data gathering ( <https://magnetospeed.com/products-v3-ballistic-chronograph> ). This chronograph mounts to the barrel of the rifle and incorporates “patented electromagnetic sensor technology” and it has a published “accuracy between 99.5% and 99.9%” (find at [www.magnetospeed.com](http://www.magnetospeed.com), Support, MagnetoSpeed General FAQ’s, Tags V3). Each round passes ~0.25 inches over the top of the chronograph and the velocities are recorded on the digital display (Figure 5).



Figure : MagnetoSpeed V3 Ballistic Chronograph



Figure : MagnetoSpeed V3 Digital Display Box

Two rifles of the same make and model were selected (make and model not provided but we can say they had 16.5in barrels). Both rifles were cleaned appropriately prior to testing. We desired 30 rounds for each of the 4 ammunitions which is a common sample size to establish significance. The 4 ammunitions (labeled A, B, C, D) were repeated for 64 shots in Rifle-1 and 44 shots in Rifle-2. The desire was to perform what is known as a Completely Randomized Design (CRD) with the same number of shots from each ammunition out of each rife on the same day. The weather did not cooperate. The first day started out at 30°F and then dropped to 20°F and the wind picked up. This forced us into a second day to test Rifle-2. The range was busy on this day, and we ran out of sunlight before we could complete all our shots. After looking at the quality of our data we found our sample size is adequate. See Table 5 for the complete set of recorded velocities. You will notice in Rifle-2 / Ammo ‘A’ / Shot #1 is crossed out. This shot for ammo ‘A’ falls almost 300 ft/s shy of the anticipated mean for this ammunition. The only explanation is that this bullet was not ammo ‘A’. It will not be used in the analysis.

Table : Recorded velocities over two days of testing.



**Selecting a Model**

The velocity data was very encouraging. Looking at Figure 6 we see four well distinguished groups as we were hoping. There is some overlap between the velocities of ammunition B and C. We will show that there is a significant difference between the two. Figure 7 displays the data split by Rifle 1 & 2. There appears to be an effect attributed rifle used but we will show later that there is not. Figure 8 shows the effect of shot number on velocity. The velocity average does not change significantly over the shot counts measured. Figure 9 displays the effect of ammunition on velocity. Figure 9 shows a clear distinction in average velocity between ammunition choice. The boxplots also show a difference in the variance of each ammunition.

Diagram

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*Figure 7: The median of Rifle-2 appears to be lower than Rifle-1.*

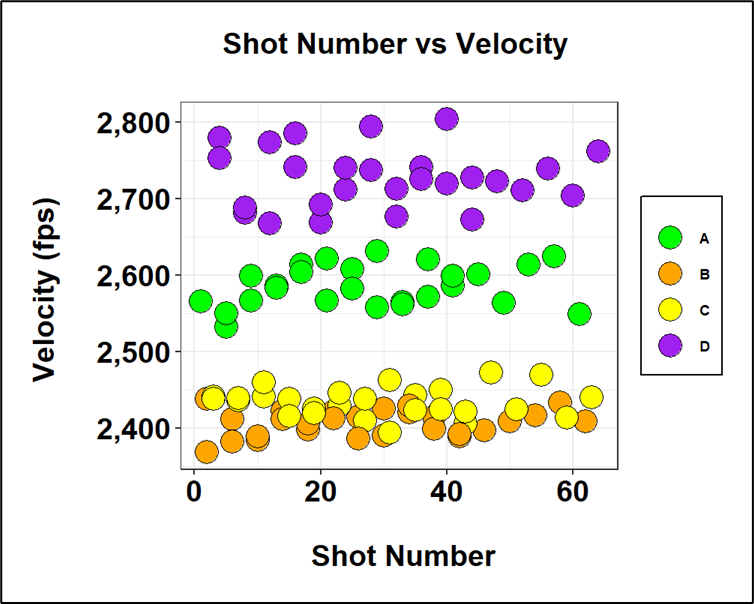
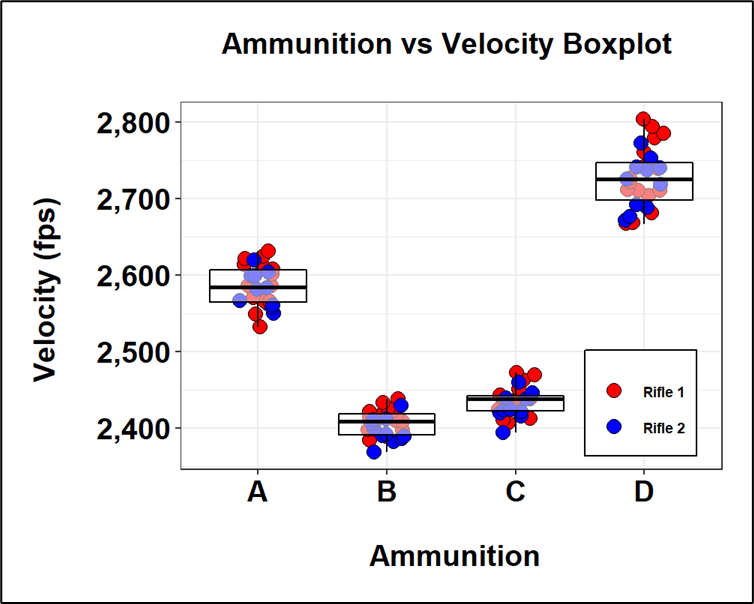
Figure : A clear distinction between ammunitions can be observed.

Figure 9: It is clear from the above boxplot that velocity and variance are unique within each group.

Figure 8: The mean value for each ammunition does not appear to be changing with shot number.

Each ammo type appears to be normally distributed, but we need to test them before we can look at their differences in variance. Figure 10 is known as a QQ Plot (Quantile-Quantile plot). Here each ammunition group is assumed to be normal. The mean and standard deviation of each group are calculated. Then each velocity value is linked to its corresponding Z-score value. This Z-score is the value on the x-axis of the normal distribution with a mean of 0 and a standard deviation of 1. The formula for determining the Z-score value is:

If the distribution of the Velocities is normal, then there will be a strong linear relationship in the plot within each ammunition group. That is what we have in Figure 10. In addition to this graphical method, we can perform a Shapiro-Wilk test. This test starts with the Null Hypothesis: the data is normal. Then it calculates what is known as the W-statistic and associated p-value. Large p-values (>0.05) mean that you cannot reject the null and the data is normal. The Shapiro-Wilk involves some lengthy calculations but again, there are many great resources on the web to learn more. If you are interested in performing the test programmatically in R, <https://www.geeksforgeeks.org/shapiro-wilk-test-in-r-programming/> provides a great explanation. Table 6 is the summary of the Shapiro-Wilk test on our dataset. You can see that each p-value is very high indicating we can be confident the ammunitions come from normally distributed populations.

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Table : The Shapiro-Wilk Test for testing a datasets normality.



Figure 10: QQ plot of Normal Z-scores with their corresponding velocity values for each group.

Looking at Figure 10 we are fairly confident that each ammunition has a distinct mean. However, ammunition B and C appear as though they might be indistinguishable from each other. We can test which group means are different using what is called the Tukey-Kramer test (TukeysHSD function in R). Performing the Tukey-Kramer test with unequal sample sizes can be referred to as a ‘myriad of tedious calculations.’ Software makes the Tukey-Kramer accessible. If you would like to perform the test in R, a great resource can be found here: <https://whitlockschluter3e.zoology.ubc.ca/Tutorials%20using%20R/R_tutorial_ANOVA.html> . The results of the Tukey-Kramer Test can be seen in Figure 11. On the x-axis are all possible pairs of ammunition. On the y-axis are the mean differences between the pairs of ammunition. The outer bounds of each box represent the 95% confidence intervals on these differences. If a confidence interval extends over zero then we can not say the means are different. In our dataset, no confidence intervals cross over the zero line. Therefore, we can say that each mean is unique at the 95% confidence level. Ammunition C and B then do not share the same mean.

**Tukey – Kramer Test of Mean Difference**

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Figure 11: Tukey-Kramer results for the mean differences in velocities between ammunition. All pairwise comparisons are significant.

We have shown that each ammunition has a normal distribution of velocities and those distributions are distinct. Now we can confidently look at the different variances and asses which one has the minimum. Table 7 contains the summary of ammunition mean, standard deviation and variance. **The minimum variance among the available ammunition is ammunition B: American Eagle 168gr OTM.**

Table : Velocity variance summary.



**Future Research**

Figure 12 shows a strong positive correlation between the mean velocity and variance. Additional ammunition should be tested to determine if velocity drives variance. We noted earlier that velocity and the ballistic coefficient have a significant covariance coefficient. DRT suggests a study on the varying performance of different BCs over Velocity. We suspect that an improved BC will yield lower variance at higher velocities.

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Figure 12: Relationship between the Mean velocity values and variance between groups.



**Conclusion**