**Ballistic Velocity Variance**

**Prepared for**

**Rabbit Creek Shooting Club**

**By Daniel Taylor**

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**Table of Contents**

**Introduction….…………………………………………………………………………………..3**

**Data Gathering………………………………………………………………………………....4**

**Sifting through the Data......................................................................5**

**Selecting a Model................................................................................6**

**Future Research...................................................................................7**

**Conclusion...........................................................................................8**

**Introduction**

We want to start by saying thank you to the Rabbit Creek Shooting Club (RCSC) for choosing DRT Consulting LLC (DRT) for this study. RCSC serves the community of Anchorage Alaska by educating youth and adults in firearms safety and recreation (Figure 1 shows the sign of RCSCs’ home range off the Seward Highway outside of Anchorage). Since DRT started serving the Anchorage community 13 years ago, we have had the privilege of providing statistical consulting for many different sporting events. However, this is the first time we have been asked to provide services related to ballistics and we are excited to show you what we have found.

RCSC is hosting a rifle competition where each shooter will make a series of 500-yard shots. Each shooter will be firing the same make and model rifle with the same factory ammunition. The goal here is to test the shooters’ ability to sight in and operate a rifle not their own while firing standard ammunition they may or may not be familiar with. RCSC has requested DRT to test the 4 available ammunitions to determine which one will have the minimum variance in velocity. This is due to RCSCs’ belief that minimizing velocity variance will minimize the variance in bullet drop. Bullet drop refers to where the bullet strikes the target on the y-axis. This minimizing will provide each competitor a fairer chance of being judged only on their skill as a shooter and not subject to the inconsistencies of the ammunition.

DRT has investigated the relevance of velocity as the primary contributor to drop and found it to be true. DRT has also identified the ammunition that provides the minimum variance to be ammunition B: ***American Eagle 168gr OTM***with a mean velocity of **2,406 ft/s** and a variance of **300 ft/s** (standard deviation = 17.3 ft/s). We also found that the velocity variance may be significantly different from rifle to rifle. One other factor, ‘*shots fired since last cleaning*’ was found to be insignificant in this test.

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Description automatically generated with low confidence

Figure : Rabbit Creek Shooting Park. (https://www.adfg.alaska.gov/index.cfm?adfg=anchoragerange.main)

**Data Gathering**

**Does Velocity Matter?**

We wanted to first investigate the importance that velocity plays in trajectory. Is there a legitimate basis for pursuing the variance in velocity? Projectile trajectories are governed by three main factors, Gravity (g), Velocity (v), and Drag (). For the purpose of this study, gravity is assumed to be a constant (g = 32.2 ft/s^2). Velocity and drag then are the varying components of a projectiles’ path. When we initially approached this project, we assumed that a simple formula could be found to relate distance traveled, flight time, etc. However, solving for projectile motion requires an rigorous application of differential equations. A simplified formula for the distance traveled by a projectile starting parallel to the ground and assuming it “never” hits the ground is:

Even this simplified formula for distance is daunting considering all bullets will eventually hit the ground, the velocity is changing over time, and drag is not linearly related to velocity above the speed of sound (1,124 feet/s).

There are several free trajectory calculators that can be downloaded to alleviate the burden of doing trajectory calculations yourself. We decided to use a free software, ‘PointBlank Ballistics’ (PBB), the download can be found here: <https://www.huntingnut.com/pointblank.html>). The software is straightforward, and the complications of differential equations are abstracted away. Figure 2 shows the graphical user interface. All other calculations and manipulations were performed in R (<https://cran.r-project.org>) or MS Excel.

Chart

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Figure : PointBlank Ballistics GUI.

The inputs of interest are Ballistic Coefficient (BC), Velocity (Vel), Weight (Wt), Altitude (Alt), Zero Distance (constant at 100-yards), and Temperature (Tmp, °F). Published values for ammunition BC, Vel, Wt can be found on the manufacturer’s website or with retailers. Table 1 has the published values for each of our tested ammunitions. The values of Alt = 200ft, Tmp = 30 °F were actual range day values.

To test the effect of changing input values on drop we set ‘Base’ equal to the ammunitions published values (Table 2 has the values used for the test, ammo ‘A’). Then each input was varied by +/- 10%. We created a table of all permutations of these values. This was done 3 variations for each of the 5 inputs means we had = 243 possible combinations. We selected a random sample of 20 from this table and used PBB to calculate bullet drop at 500-yards. Table 3 has the complete table for analysis of inputs on bullet drop.

Table : Published specifications for selected ammunition.



Table : Ammunition 'A' Input Parameters for test.



Table : Sampled data from Ammo 'A' permutation table.



The sampled data revealed the impact of each of the inputs on the bullet drop. Figure 3 shows the impact of Velocity and BC on bullet drop. Drop is inversely proportional to both Velocity and BC. Another observation is Velocity and BC do not appear to be independent. As the Velocity increases the effect of BC appear to decrease.

Chart, scatter chart

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Figure : Effect of Velocity and BC on Bullet Drop calculation.

We tested the significance of each input using a linear model and a method called Analysis of Variance (ANOVA). These are powerful statistical tools that allow us to make decisions on what model will be best for our analysis. We started out by assuming that:

1. Bullet drop is equal to a linear combination of all the inputs.
2. All of the errors are normally distributed.
3. The variance is constant.

There are several tests available to ensure each of these assumptions holds true. For our purposes, we will let the assumptions stand. Below is the linear model we incorporated:

Each of the β’s are coefficients for the independent variables. is the intercept term. Notice also that we are assuming an interaction (Vel \* BC) between velocity and the ballistic coefficient. When we run the model in R (using the lm() function) our coefficients are found to be:

Now that we have the linear model, we can find out which variables are “significant”. For our purposes, we want to be 99% confident that a variable is informative before we employ it in our model (statisticians are always looking for the most *parsimonious* model). We will use ANOVA to establish significance.

ANOVA begins with a Hypothesis that none of the inputs to the model matter. Then for each input we proceed to get the Degrees of Freedom (Df), Mean Squared Error (MSE), F-Value, and the F distributions tail probability given the degrees of freedom of the input and the residuals. This process can be challenging to understand. A web search of ANOVA is a great start to understanding the workflow. I suggest starting at the site Guru99 (<https://www.guru99.com/r-tutorial.html>) and selecting the R ANOVA Tutorial under the Data Analysis section. Table 4 is the ANOVA summary table. Observe in the ANOVA table that we do not have sufficient evidence to conclude that Velocity, Ballistic Coefficient and their interaction are insignificant at the 99% level. Therefore, pursuing the minimization of velocity variance is important. While the pursuit of maximizing the Ballistic Coefficient is also significant, that is outside the scope of this study.

Table : ANOVA table for test of significance of model inputs.



How will this benefit people

* What is the goal of the study
* Describe the response variable

Data Handling

* How errors are handled.
* What program was used.
* Changes to the data set.
* Formatting issues

Explain the model

* Justify the analysis chosen.
* Explain theory if it adds to the explanation
* Define the alpha/confidence level

Model Validation

* Residuals
* Confusion matrix
* Prediction accuracy
* Standard errors
* Effect graphs

Searched for more complex effects

* Interactions
* Transformations
* Curvature

How independent variables affect the response

* Graphs of effects
* Interactions / transformations explained
* Accuracy of the results.
* Variables removed from the study

Graphs

* Labelled adequately.
* Easy to understand
* Explanation of what is seen in the graph
* Helps explain the results

Output Explained

* Everything in the report is referenced and discussed and explained with a connection to how it answers the question.

Answer the question

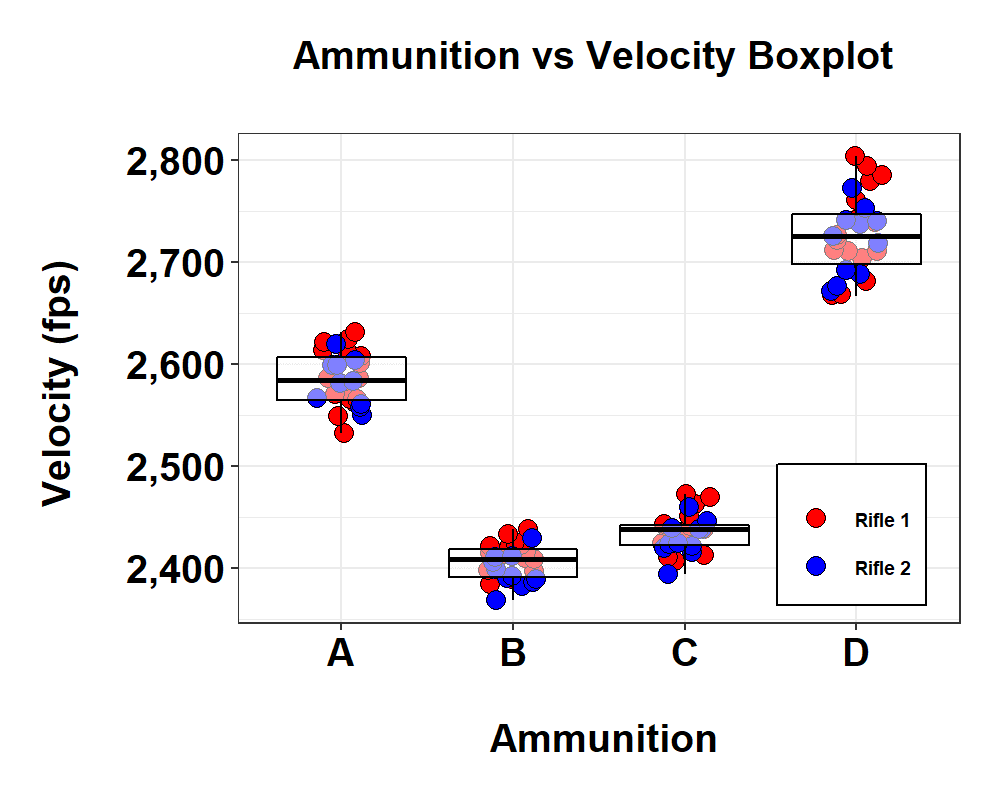
* What the client wanted to get from your paper.
* Explain the results and summarize the findings easily

Conclusion

* Summarized the findings of the report
* How this will benefit people.
* Where future research could go.

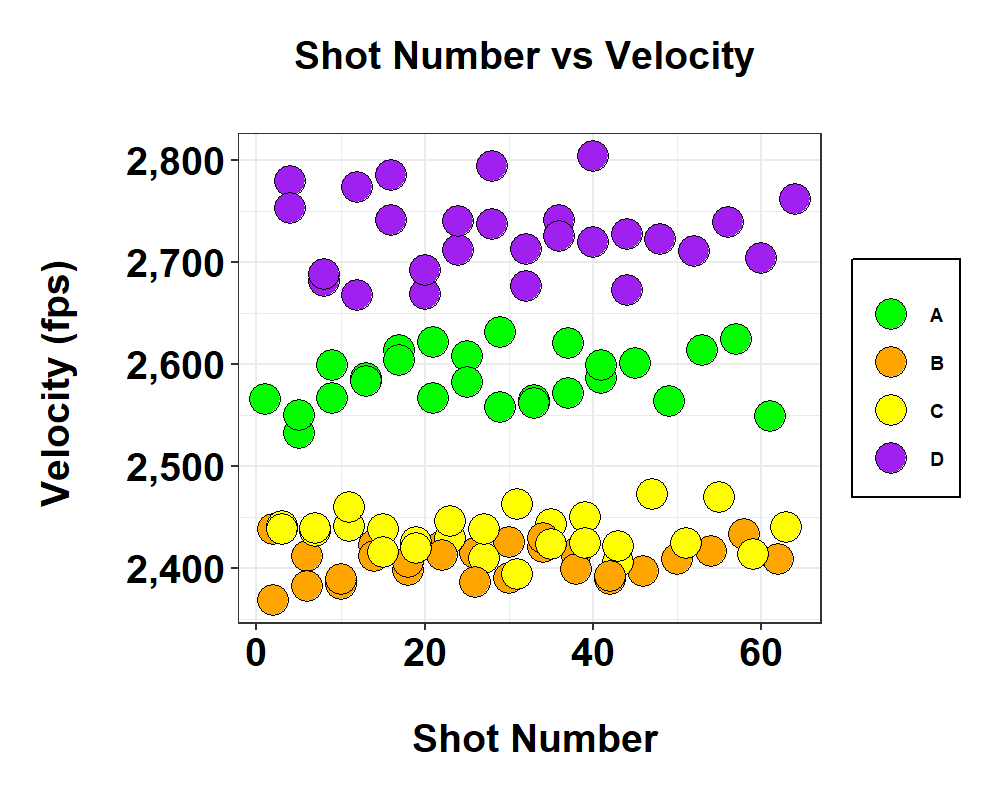


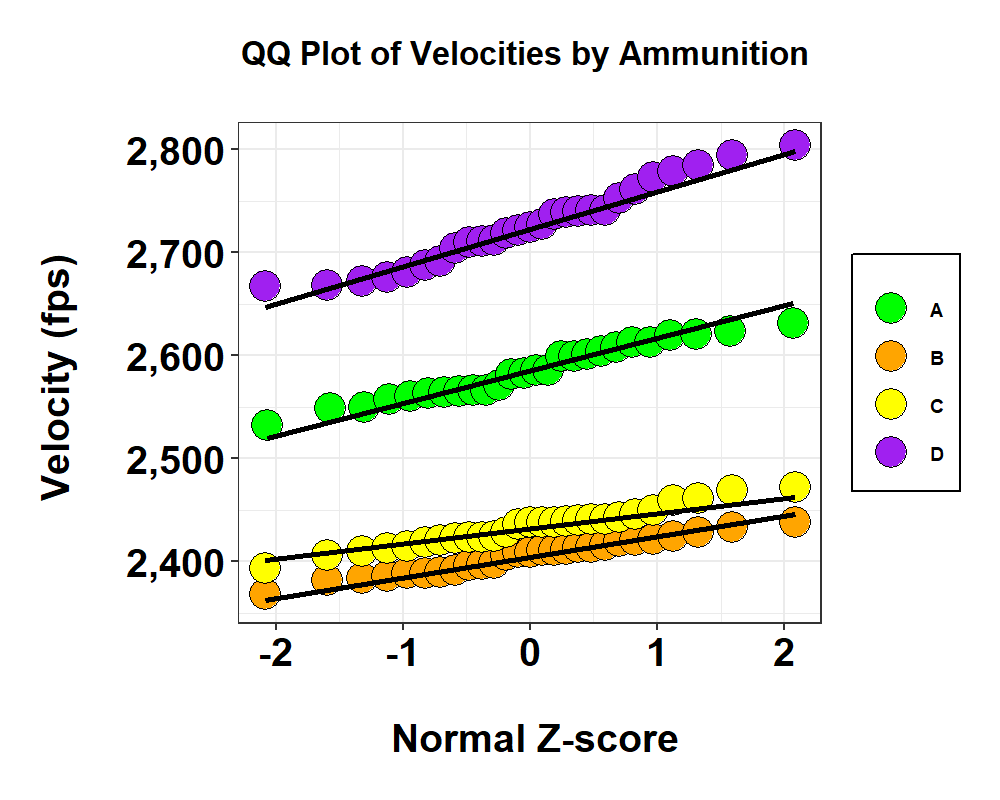




Diagram

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**Introduction**

Data Gathering